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A study on quantum decoherence phenomena with three generations of neutrinos

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Abstract

Using the open quantum system approach applied to the neutrino system, we derive three generations neutrino probability formulae considering the oscillation induced by mass plus quantum decoherence contributions. The introduction of these dissipative effects is done through the quantum dynamical semigroup formalism. In addition to the theoretical interest of the approach, at least from the completeness point of view, this extension of the formalism to the three flavors, provide us with a direct application: we can analyze qualitatively the consistency of the two generation pure decoherence solution to the atmospheric neutrino problem, accommodated within this enlarged scheme, with the mean tendencies observed for some of the current neutrino experimental data. This study was performed based on different choices of the 3×3 mixing matrix selected in order to adjust the $P_{\nu_\mu \rightarrow \nu_\mu}$ to the same form it has for the decoherence solution in two generations. Our qualitative tests for decoherence with three neutrinos show a clear incompatibility between neutrino data and the theoretical expectations.

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I. INTRODUCTION

In the open quantum system approach [1], the evolution of a system interacting with an environment is described in an effective way, that is, the interaction with the environment is incorporated in the description of the evolution of the system. In general, the effects produced by the interaction cause dissipation and irreversibility. This treatment was originally developed for quantum optics [2], in order to take into account the system-reservoir (environment) interaction, and it has already been applied to elementary particle systems. Recently, it has also been used in the study of two neutrino oscillations [3,4], modifying the well known oscillation probability, due to the presence of dissipative effects. One very interesting feature of the oscillation probability in this situation is that even in the case that neutrinos are massless, we can have neutrino flavor conversion [4–6]. It has been pointed out that this mechanism is able to explain well, in the context of two generations, the atmospheric neutrino data collected by the Super-Kamiokande (SK) experiment [7], as long as the damping parameter is $\propto 1/E_\nu$, E_ν being the neutrino energy.

The main goal of this paper is to extend this formalism of mass, mixing and quantum decoherence to three generations. This is rather well motivated since nowadays the atmospheric and solar neutrino observations [8,9] can only be explained by the introduction of some neutrino flavor conversion mechanism which must be understood, specially after the recent impressive SNO results [10], in terms of three generations. Therefore, this extension will be very useful, because it will permit us to study the decoherence contributions on top of the oscillation induced by mass (OIM) in a complete three neutrino context.

We have developed this three neutrino formalism using the powerful technique of quantum dynamical semigroups [11,1], which makes our analysis independent of any hypothesis about the interaction between the neutrino system and the environment. Quantum open systems can also be treated using the master equation formalism [2], but in this case, a previous knowledge about the interaction with the environment is required. This is an important point since it is not clearly established which is the origin of the pervasive medium, the most likely possibilities at the moment are quantum gravity effects described by strings at low energy range. In this way, our results are broader from the phenomenological point of view. Based on motivations coming from the master equation formalism using the weak coupling limit in two generations [3,4], we have casted the most relevant decoherence effects in a diagonal dissipation matrix, as a result we have obtained manageable probability expressions.

Additionally, as a phenomenological application of this three neutrino extension for the decoherence phenomena, we have performed a qualitative study to inspect the agreement between the average behaviour of relevant neutrino data and the decoherence solution to the atmospheric neutrino anomaly in two generations, embodied within this enlarged neutrino framework. We have done this study assuming different alternatives for the introduction of the two generations decoherence solution in our three neutrino scheme. Basically we have divided the study in two parts.

On the first part we use only PD to fully describe three neutrino flavor conversions, and on the second we deal with a hybrid case, where we assume decoherence plus OIM take place in the $\nu_e \rightarrow \nu_{\mu(\tau)}$ sector.

This paper is organized as follows. In Sec. II we describe the quantum dynamical semigroup formalism for a three level quantum system. In Sec. III we apply this formalism for the neutrino system, explicitly calculating the survival and conversion probabilities among neutrino flavors under the influence of decoherence effects. In Sec. IV we analyze the consistence of the decoherence solution to the atmospheric neutrino problem in the three generation framework. Finally in Sec. V we present our conclusions.

II. QUANTUM DYNAMICAL SEMIGROUPS AND THREE LEVEL SYSTEMS

Hamiltonian evolution, that is, the time evolution of a physical system described by the Schrödinger equation in the case of pure ensembles, or by the Liouville equation for mixed ones, is a characteristic of systems isolated from their surroundings. The time evolution of an isolated quantum system with Hamiltonian H is given by the continuous group of unitary transformations $U_t = e^{iHt}$, where t is the time. From the mathematical point of view, the existence of the inverse of the infinitesimal generator H , a consequence of the algebraic structure of a group, gives rise to *reversible processes* in Hamiltonian systems.

We know that the time evolution of a quantum open system is characterized by the presence of dissipative effects which, in turn, give rise to the *irreversible nature* of the evolution. So, if a given family of transformations should be responsible for the evolution in time of a quantum open system, this family will certainly not be a group. A rigorous mathematical treatment of quantum open systems is provided by the so called quantum dynamical semigroups [11,1].

The evolution generated by the operators in these semigroups has the property of being forward in time, as a consequence of the lack of inverse in a semigroup. Physically, this property can be interpreted as the existence of an arrow of time which in turn makes possible the connection with thermodynamics via an entropy.

According to Ref. [12], if \mathcal{H} is the Hilbert space of a given open quantum system, and $\mathcal{B}(\mathcal{H})$ the space of bounded operators acting on \mathcal{H} , the infinitesimal generator \mathcal{L} , defined through its action on the density matrix $\rho(t)$, is given by

$$\mathcal{L}\rho(t) = \frac{\partial \rho(t)}{\partial t} = -i[H_{\text{eff}}, \rho(t)] + \frac{1}{2} \sum_j \left([A_j, \rho(t)A_j^\dagger] + [A_j\rho(t), A_j^\dagger] \right), \quad (1)$$

where $H_{\text{eff}} = H + H_d$ is the “effective” Hamiltonian of the system, H being its free Hamiltonian and H_d accounts for possible additional dissipative contributions which can be incorporated to H , in other words, which can be put in the Hamiltonian form. A_j is a sequence of bounded operators of \mathcal{H} ($A_j \in \mathcal{B}(\mathcal{H})$) satisfying $\sum_j A_j^\dagger A_j \in \mathcal{B}(\mathcal{H})$.

The first term in Eq. (1) constitutes the Hamiltonian part of the evolution, whereas the second term is responsible for the irreversible (non-hamiltonian) nature of an open system evolving in time.

Therefore, if we interpret the last expression as an effective equation describing the reduced dynamics of an open system interacting with a certain “environment”, the second term in Eq. (1), in a certain sense, represents the interaction between the open system and the mentioned environment. However, the generator \mathcal{L} does not depend on a particular interaction, being constructed based on very general hypothesis about the time evolution, that is, irreversible dynamics, conservation of probability and a less intuitive hypothesis known as complete positivity [12,1].

For a N-level system it is possible to construct an explicit parameterization of Eq. (1), provided that a suitable basis of $\mathcal{B}(\mathcal{H})$, viewed as a vector space, is chosen [13]. From now on, we will restrict ourselves to a three-level system, whose evolution is governed by Eq. (1) and our approach will essentially follow Ref. [3]. The bounded operators in $\mathcal{B}(\mathcal{H})$ can be represented by 3×3 matrices of $M_3(\mathbf{C})$, which in turn, can be generated by a basis $\{F_\mu, \mu = 0, 1, \dots, 8\}$, endowed with the scalar product $\langle F_\mu, F_\nu \rangle = \text{Tr}(F_\mu^\dagger F_\nu)$ and satisfying

$$\langle F_\alpha, F_\beta \rangle = \frac{1}{2} \delta_{\alpha\beta}. \quad (2)$$

We adopt here the standard basis of hermitian matrices

$$F_0 = \frac{1}{\sqrt{6}} \mathbf{1}_3, \quad F_i = \frac{1}{2} \Lambda_i \quad (i = 1, \dots, 8), \quad (3)$$

where the Λ_i are the Gell-Mann matrices [14]¹. Using this choice, F_i 's satisfy the Lie algebra

$$[F_i, F_j] = i \sum_k f_{ijk} F_k, \quad 1 \leq i, j, k \leq 8, \quad (4)$$

where f_{ijk} are the structure constants of $SU(3)$. Expanding the operators in Eq. (1) in the adopted basis we get²

$$H_{\text{eff}} = \sum_\mu h_\mu F_\mu, \quad \rho = \sum_\mu \rho_\mu F_\mu, \quad A_j = \sum_\mu a_\mu^{(j)} F_\mu. \quad (5)$$

As pointed out in Ref. [15], the hermiticity of A_j is a condition that assures the increasing with time of the von Neuman entropy $S = -\text{Tr}(\rho \ln \rho)$, with this choice, the second term on the right hand side of Eq. (1) can be written as

¹There is a standard method of constructing a set of matrices satisfying Eq. (2) in $M_n(\mathbf{C})$ [11].

²From now on, Greek indices will range from 0 to 8, while Latin indices will range from 1 to 8, unless otherwise stated.

$$\frac{1}{2} \sum_j \{[A_j, \rho A_j] + [A_j \rho, A_j]\} = \sum_{\mu, \nu} L_{\mu\nu} \rho_\mu F_\nu, \quad (6)$$

where

$$L_{\mu 0} = L_{0\mu} = 0, \quad L_{ij} = \frac{1}{2} \sum_{k,l,m} (\vec{a}_m \cdot \vec{a}_k) f_{iml} f_{lkj}, \quad (7)$$

and in the last equation we have introduced the vectors $\vec{a}_\mu = \{a_\mu^{(1)}, a_\mu^{(2)}, \dots, a_\mu^{(8)}\}$ of \mathbf{R}^8 with the usual scalar product $\vec{a}_\mu \cdot \vec{a}_\nu = \sum_j a_\mu^{(j)} a_\nu^{(j)}$. $L_{\mu\nu}$ is a real³, symmetric matrix, defined according to Eqs. (6) and (7). The elements of $L_{\mu\nu}$ are not totally arbitrary, but satisfy some relations due to the presence of the scalar product, which in turn, should obey the Cauchy-Schwartz inequality. Introducing the remaining expansions of Eq. (5) into Eq. (1) we get finally

$$\dot{\rho}_\mu = \sum_{i,j} h_i \rho_j f_{ij\mu} + \sum_\nu L_{\mu\nu} \rho_\nu \quad \mu = 0, \dots, 8. \quad (8)$$

The system of first order differential equations above describes the time evolution of a three-level quantum open system. Such an evolution is governed by the laws of quantum dynamical semigroups. The physical processes exhibit an arrow of time as a consequence of the monotonical increase of the von Neumann entropy as a function of time. The evolution also allows the interpretation of the eigenvalues of $\rho(t)$ at any instant of time as the probability of finding the open system in the eigenstate associated with the eigenvalue.

As already mentioned, the theoretical approach provided by quantum dynamical semigroups is a very general one to treat open quantum systems in the sense that no explicit hypothesis has to be made about the possible interactions causing the loss of quantum coherence. On the other hand, there is another approach to deal with systems of the same nature, in which a well defined form of the interaction ought to be provided in order to derive the reduced dynamics of the open quantum system. This approach is known as the master equation formalism and its rigorous mathematical formulation can be found in a series of papers [16,17,13,18–22]. Furthermore, it is possible to show that the master equation formalism applied to an open system weakly coupled to some environment leads to an evolution equation for the reduced density matrix of the open system compatible of that obtained from Eq. (1) [11], in other words, in this situation, the master equation and the quantum dynamical semigroup formalism are equivalent.

Coming back to Eq. (8), we see that the differential equation for the $\rho_0(t)$ component and conservation of probability imply $\rho_0(t) = \sqrt{2/3}$. The remaining differential equations have the form

³The real nature of its entries can be easily deduced from the hermiticity of A_j .

$$\dot{\rho}_k = \sum_j \left(\sum_i h_i f_{ijk} + L_{kj} \right) \rho_j = \sum_j M_{kj} \rho_j, \quad (9)$$

which in the matrix form can be written as ⁴

$$\dot{\varrho} = \mathbf{M}\varrho, \quad (10)$$

so that the formal solution is given by

$$\varrho(t) = e^{\mathbf{M}t} \varrho(0). \quad (11)$$

If $\{\lambda_1, \dots, \lambda_8\}$ and $\{\mathbf{v}_1, \dots, \mathbf{v}_8\}$ are the set of eigenvalues and eigenvectors of \mathbf{M} , respectively, the matrix \mathbf{D} , with entries $D_{ij} = (\mathbf{v}_i)_j$, diagonalizes \mathbf{M} ,

$$\mathbf{M}' = \mathbf{D}^{-1} \mathbf{M} \mathbf{D} = \text{diag}(\lambda_1, \dots, \lambda_8). \quad (12)$$

Applying the inverse transformation, $e^{\mathbf{M}t} = \mathbf{D} e^{\mathbf{M}'t} \mathbf{D}^{-1}$, we get

$$\rho_i(t) = \sum_{k,j} e^{\lambda_k t} D_{ik} D_{kj}^{-1} \rho_j, \quad (13)$$

so that the complete solution of the system of differential equations (8) is equivalent to a problem of eigenvalues and eigenvectors. In the case of a matrix of order 8, with all non null entries, the solution is too complicated to allow for direct physical interpretations.

III. DISSIPATIVE EFFECTS AND NEUTRINOS

In this section we present an expression for the probability of neutrino flavor conversion assuming that the dynamics responsible by this process is constituted by the already known standard OIM mechanism, as well as by dissipative effects according to quantum dynamical semigroups.

A. The probability of conversion

The hamiltonian H for a free relativistic neutrino with momentum \mathbf{p} and rest mass m is given by

$$H = \sqrt{\mathbf{p}^2 + m^2}, \quad (14)$$

so that in the basis of mass eigenstates

⁴Note that $\varrho \neq \rho$, the first being a column vector with components ρ_k .

$$\mathbf{p}|\nu_k\rangle = \mathbf{p}_k|\nu_k\rangle \quad \text{and} \quad m|\nu_k\rangle = m_k|\nu_k\rangle, \quad (15)$$

and we can write for relativistic neutrinos ($p = |\mathbf{p}|$),

$$H \sim p^2 + \frac{m^2}{2p} \implies \langle \nu_k | H | \nu_l \rangle = \delta_{kl} \left(p_k + \frac{m_k^2}{2p_k} \right). \quad (16)$$

The expansion of H in the basis F_μ is

$$H = \frac{1}{2p} \sqrt{\frac{2}{3}} \left(6p^2 + \sum m^2 \right) F_0 + \frac{1}{2p} \left(\Delta m_{12}^2 \right) F_3 + \frac{1}{2\sqrt{3}p} \left(\Delta m_{13}^2 + \Delta m_{23}^2 \right) F_8, \quad (17)$$

where $\sum m^2 = m_1^2 + m_2^2 + m_3^2$ and $\Delta m_{ij}^2 = m_i^2 - m_j^2$, $i, j = 1, 2, 3$.

As already mentioned at the end of the preceding section, an analysis considering the most general form of \mathbf{M} would certainly not provide information subject to direct physical interpretation. From this point of view, it is reasonable to begin the analysis introducing simplifications which allow a direct physical interpretation of the results. Of course, these simplifications cannot be completely arbitrary, because all the physics depends on them. In this way, motivations coming from the formalism of master equation, in the so called weak coupling limit, applied to the problem of neutrino flavor conversion in the two generation case [4], lead us to assume a diagonal matrix $L_{\mu\nu}$, that is,

$$[L_{\mu\nu}] = \text{diag}(0, -\gamma_1, -\gamma_2, -\gamma_3, -\gamma_4, -\gamma_5, -\gamma_6, -\gamma_7, -\gamma_8), \quad (18)$$

where the diagonal elements are given by

$$\gamma_i = L_{ii} = -\frac{1}{2} \sum_{k,l,m} (\vec{a}_m \cdot \vec{a}_k) f_{iml} f_{ikl}. \quad (19)$$

In the case of two generations, the diagonal form of $L_{\mu\nu}$ can be deduced via the master equation formalism in the weak coupling limit, provided that a gas of quanta, satisfying infinite statistics (for example a D0-brane) is adopted as the dissipative medium. These quanta should also be in thermodynamic equilibrium at a finite temperature $\beta = 1/M$, where M defines an energy scale at which the dissipative effects are believed to become important (possibly the Planck scale if the effects have quantum gravity as their source). The diagonal form is finally obtained if one further imposes the condition of entropy increase for finite β , for details see Ref. [4]. We stress that here Eq. (18) is just an *Ansatz* motivated by the results in the two generation case.

Using this parameterization of $L_{\mu\nu}$ we can construct the matrix \mathbf{M} defined in Eq. (9), and its eigenvalues and eigenvectors can be obtained. The explicit form of \mathbf{M} , its eigenvalues and associated eigenvectors can be found in Appendix A.

If at $t = 0$ a neutrino of flavor ν_α is produced, the probability of its conversion, after a time t , to a flavor ν_β can be written as

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t) = \text{Tr}[\rho^\alpha(t)\rho^\beta] = \frac{1}{3} + \frac{1}{2} \sum_{i,k,j} e^{\lambda_k t} D_{ik} D_{kj}^{-1} \rho_j^\alpha(0) \rho_i^\beta. \quad (20)$$

In a more explicit form, the probability of conversion can be written as

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta}(t) = & \frac{1}{3} + \frac{1}{2} \left\{ \left[\left(\rho_1^\alpha \rho_1^\beta + \rho_2^\alpha \rho_2^\beta \right) \left(\frac{e^{-\frac{\Omega_{12}t}{2}} + e^{\frac{\Omega_{12}t}{2}}}{2} \right) \right. \right. \\ & - \left. \left(\frac{2\Delta_{12} \left(\rho_1^\alpha \rho_2^\beta - \rho_2^\alpha \rho_1^\beta \right) + \Delta\gamma_{12} \left(\rho_1^\alpha \rho_1^\beta - \rho_2^\alpha \rho_2^\beta \right)}{\Omega_{12}} \right) \left(\frac{e^{-\frac{\Omega_{12}t}{2}} - e^{\frac{\Omega_{12}t}{2}}}{2} \right) \right] e^{-\frac{1}{2}(\gamma_1 + \gamma_2)t} \\ & + \left[\left(\rho_4^\alpha \rho_4^\beta + \rho_5^\alpha \rho_5^\beta \right) \left(\frac{e^{-\frac{\Omega_{13}t}{2}} + e^{\frac{\Omega_{13}t}{2}}}{2} \right) \right. \\ & - \left. \left(\frac{2\Delta_{13} \left(\rho_4^\alpha \rho_5^\beta - \rho_5^\alpha \rho_4^\beta \right) + \Delta\gamma_{45} \left(\rho_4^\alpha \rho_4^\beta - \rho_5^\alpha \rho_5^\beta \right)}{\Omega_{13}} \right) \left(\frac{e^{-\frac{\Omega_{13}t}{2}} - e^{\frac{\Omega_{13}t}{2}}}{2} \right) \right] e^{-\frac{1}{2}(\gamma_4 + \gamma_5)t} \\ & + \left[\left(\rho_6^\alpha \rho_6^\beta + \rho_7^\alpha \rho_7^\beta \right) \left(\frac{e^{-\frac{\Omega_{23}t}{2}} + e^{\frac{\Omega_{23}t}{2}}}{2} \right) \right. \\ & - \left. \left(\frac{2\Delta_{23} \left(\rho_6^\alpha \rho_7^\beta - \rho_7^\alpha \rho_6^\beta \right) + \Delta\gamma_{67} \left(\rho_6^\alpha \rho_6^\beta - \rho_7^\alpha \rho_7^\beta \right)}{\Omega_{23}} \right) \left(\frac{e^{-\frac{\Omega_{23}t}{2}} - e^{\frac{\Omega_{23}t}{2}}}{2} \right) \right] e^{-\frac{1}{2}(\gamma_6 + \gamma_7)t} \\ & \left. + e^{-\gamma_3 t} \rho_3^\alpha \rho_3^\beta + e^{-\gamma_8 t} \rho_8^\alpha \rho_8^\beta \right\}, \quad (21) \end{aligned}$$

where $\Delta\gamma_{ij} = \gamma_j - \gamma_i$ and the definition of the new variables Ω_{ij} can be found in Appendix A.

It is easy to verify that: Eq. (21) exhibits conservation of probability at any instant of time; reducing the number of dimensions from 3 to 2, we get the results for decoherence in two generations [3]; and in the limit $\gamma_i \rightarrow 0$ ($1 \leq i \leq 8$), the standard OIM mechanism is recovered.

IV. PHENOMENOLOGICAL APPLICATION

In this section we will use the just developed three neutrino framework for OIM mechanism plus the decoherence effects, to analyze the robustness of the decoherence solution to the atmospheric neutrino problem in two generations [7] envisaged within this enlarged scheme. This will be done for different selections of the mixing matrix, first for PD and second for decoherence plus masses and mixing.

A. Pure Decoherence

In order to obtain the probabilities that correspond to the PD case it is enough to eliminate the usual oscillatory terms. This can be easily done by setting the

mixing matrix U equal to the unity matrix. Applying this to Eq. (21) we get the following expressions:

$$P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{PD}}(t) = \frac{1}{3} + \frac{1}{2}(e^{-\gamma_3 t} \rho_3^\alpha \rho_3^\beta + e^{-\gamma_8 t} \rho_8^\alpha \rho_8^\beta), \quad (22)$$

where $\alpha, \beta = e, \mu, \tau$, so consequently

$$P_{\nu_e \rightarrow \nu_e}^{\text{PD}}(t) = \frac{1}{3} + \frac{1}{2}e^{-\gamma_3 t} + \frac{1}{6}e^{-\gamma_8 t}, \quad (23)$$

$$P_{\nu_e \rightarrow \nu_\mu}^{\text{PD}}(t) = \frac{1}{3} - \frac{1}{2}e^{-\gamma_3 t} + \frac{1}{6}e^{-\gamma_8 t}, \quad (24)$$

$$P_{\nu_e \rightarrow \nu_\tau}^{\text{PD}}(t) = \frac{1}{3} - \frac{1}{3}e^{-\gamma_8 t}, \quad (25)$$

$$P_{\nu_\mu \rightarrow \nu_\mu}^{\text{PD}}(t) = \frac{1}{3} + \frac{1}{2}e^{-\gamma_3 t} + \frac{1}{6}e^{-\gamma_8 t}, \quad (26)$$

$$P_{\nu_\mu \rightarrow \nu_\tau}^{\text{PD}}(t) = \frac{1}{3} - \frac{1}{3}e^{-\gamma_8 t}. \quad (27)$$

Despite the fact that mixing is not included, we can observe from these probabilities, that it is possible to have non-null neutrino flavor conversion only taking into account the decoherence phenomena. Indeed, these results were expected, since the same was obtained in two generations [4].

The atmospheric neutrino data can be interpreted well using the PD phenomena in the two generation framework [7]. Thus, we can investigate if the pure decoherence mechanism extrapolated to three generations is still able to explain the current data. A qualitative analysis of the behavior of the above probabilities in the present data context can clearly demonstrate that this is not so. The explanation goes as follows.

Since we aim to test the decoherence solution to the atmospheric neutrino data in the three generations scheme, we will have to consider the damping parameters with the dependence $\gamma_j \rightarrow \gamma_j/E_\nu$, in accordance with the two generations assumption. Thus, the probabilities defined in Eqs. (23)-(27) will be sensitive only to the ratio L/E_ν ⁵ similar to vacuum case for the OIM mechanism. Here L is the distance between the source of neutrinos and the detector. Note also that these probabilities will not be modified by the presence of matter, since there is no mixing involved.

The conjunction of all of these properties of the probabilities with the fact that $P_{\nu_e \rightarrow \nu_e} = P_{\nu_\mu \rightarrow \nu_\mu}$, makes the three neutrino PD mechanism incompatible with the actual neutrino data. One can exemplify this inconsistency by confronting the results of the neutrino experiments CHOOZ [23] and K2K [24] with our theoretical predictions. Both experiments have similar L/E_ν , but their results for $P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$ and $P_{\nu_\mu \rightarrow \nu_\mu}$ are not compatible with our theoretical expectation. The CHOOZ experiment with

⁵In our formulae you can make the substitution $t \leftrightarrow L$ (for $c=1$).

$\langle L/E_\nu \rangle \approx 1000/3$ (m/MeV) observed $\langle P_{\nu_e \rightarrow \nu_e} \rangle \approx 1$ ⁶ and the K2K experiment with $\langle L/E_\nu \rangle \approx 250/1.3$ (km/GeV) has observed events compatible with $\langle P_{\nu_\mu \rightarrow \nu_\mu} \rangle \approx 0.7$. Therefore the relation $P_{\nu_\mu \rightarrow \nu_\mu} = P_{\nu_e \rightarrow \nu_e}$ given by our probabilities, is in contradiction with the results of CHOOZ and K2K.

Another example that reinforces this contradiction is the multi-GeV SK data sample [25] in combination with the CHOOZ constraint. In order to illustrate this fact, we consider the SK normalized e -like and μ -like event samples, which can be computed as follows

$$R_\mu = \langle P_{\nu_\mu \rightarrow \nu_\mu} \rangle + \frac{1}{r} \langle P_{\nu_e \rightarrow \nu_\mu} \rangle, \quad (28)$$

$$R_e = \langle P_{\nu_e \rightarrow \nu_e} \rangle + r \langle P_{\nu_\mu \rightarrow \nu_e} \rangle, \quad (29)$$

where R_μ and R_e are the normalized μ -like and e -like event samples defined as the ratios of the expected number of events considering neutrino conversion over the expected number of events without neutrino flavor conversion, r is the original proportion between the muon and electron neutrino fluxes. We can observe that for ranges of $\langle L/E_\nu \rangle \approx 1000/3$ compatible with CHOOZ and corresponding to $\cos \theta_{\text{zenith}} \sim -0.22$ ⁷ in the multi-GeV sample, $\langle E_\nu \rangle \approx 10$ GeV [26], $R_\mu = 0.6 - 0.7$ and $R_e = 1$. Thus, crossing this information with the CHOOZ bound, we find that $\langle P_{\nu_\mu \rightarrow \nu_e} \rangle = \langle P_{\nu_e \rightarrow \nu_\mu} \rangle \approx 0$ (in fact, for a real mixing matrix, $P_{\nu_\alpha \rightarrow \nu_\beta} = P_{\nu_\beta \rightarrow \nu_\alpha}$ as can be seen from Eq. (21)) and consequently $\langle P_{\nu_\mu \rightarrow \nu_\mu} \rangle \approx 0.6 - 0.7$, which is again in conflict with our theoretical prediction.

B. Decoherence \oplus Oscillation

Here we will study two different cases also admitting the presence of the oscillatory terms in the probability expressions, therefore we have here a mixed situation of decoherence \oplus oscillation (D \oplus O). In both cases the oscillation will be included through non-null mixing matrix elements, which connect ν_e with ν_μ or ν_e with ν_τ . This is chosen in order to preserve the possibility to explain the solar neutrino data through of the OIM mechanism. In addition, another fact in our strategy of analysis will be to adjust the parameters in the $P_{\nu_\mu \rightarrow \nu_\mu}$, the leading channel for the explanation of the atmospheric data, to be similar to its form in the two generations decoherence solution to the atmospheric neutrino problem.

⁶The probabilities defined in Eq. (21) satisfy $P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha} \equiv P_{\nu_\alpha \rightarrow \nu_\alpha}$.

⁷The relation between the $\cos \theta_{\text{zenith}}$ and L is given by $L = \sqrt{(R_T + h)^2 - (R_T \sin \Theta_Z)^2} - R_T \cos \theta_{\text{zenith}}$, where R_T is the earth radius and h is the height of the neutrino production point.

When we introduce non-zero masses for the neutrinos, two different situations may arise according to the magnitudes of Δ_{ij} and $\Delta\gamma_{kl}$: either $2|\Delta_{ij}| \geq |\Delta\gamma_{kl}|$ or $2|\Delta_{ij}| < |\Delta\gamma_{kl}|$. In our analysis we will assume the first situation, where Ω_{ij} is imaginary so that Eq. (21) takes the form

$$\begin{aligned}
P_{\nu_\alpha \rightarrow \nu_\beta}(t) = & \frac{1}{3} + \frac{1}{2} \left\{ \left[\left(\rho_1^\alpha \rho_1^\beta + \rho_2^\alpha \rho_2^\beta \right) \cos \left(\frac{|\Omega_{12}|t}{2} \right) \right. \right. \\
& + \left. \left(\frac{2\Delta_{12} \left(\rho_1^\alpha \rho_2^\beta - \rho_2^\alpha \rho_1^\beta \right) + \Delta\gamma_{12} \left(\rho_1^\alpha \rho_1^\beta - \rho_2^\alpha \rho_2^\beta \right)}{\Omega_{12}} \right) \sin \left(\frac{|\Omega_{12}|t}{2} \right) \right] e^{-\frac{1}{2}(\gamma_1 + \gamma_2)t} \\
& + \left[\left(\rho_4^\alpha \rho_4^\beta + \rho_5^\alpha \rho_5^\beta \right) \cos \left(\frac{|\Omega_{13}|t}{2} \right) \right. \\
& + \left. \left(\frac{2\Delta_{13} \left(\rho_4^\alpha \rho_5^\beta - \rho_5^\alpha \rho_4^\beta \right) + \Delta\gamma_{45} \left(\rho_4^\alpha \rho_4^\beta - \rho_5^\alpha \rho_5^\beta \right)}{\Omega_{13}} \right) \sin \left(\frac{|\Omega_{13}|t}{2} \right) \right] e^{-\frac{1}{2}(\gamma_4 + \gamma_5)t} \\
& + \left[\left(\rho_6^\alpha \rho_6^\beta + \rho_7^\alpha \rho_7^\beta \right) \cos \left(\frac{|\Omega_{23}|t}{2} \right) \right. \\
& + \left. \left(\frac{2\Delta_{23} \left(\rho_6^\alpha \rho_7^\beta - \rho_7^\alpha \rho_6^\beta \right) + \Delta\gamma_{67} \left(\rho_6^\alpha \rho_7^\beta - \rho_7^\alpha \rho_6^\beta \right)}{\Omega_{23}} \right) \sin \left(\frac{|\Omega_{23}|t}{2} \right) \right] e^{-\frac{1}{2}(\gamma_6 + \gamma_7)t} \\
& \left. + e^{-\gamma_3 t} \rho_3^\alpha \rho_3^\beta + e^{-\gamma_8 t} \rho_8^\alpha \rho_8^\beta \right\}. \tag{30}
\end{aligned}$$

In the case $2|\Delta_{ij}| < |\Delta\gamma_{kl}|$, we would have an analogous expression, but with hyperbolic sines and cosines.

1. Mixing in the ν_e - ν_μ sector

In this case we switch on only the non-null mixing matrix elements in the ν_e - ν_μ sector, so that the mixing matrix is defined as

$$U = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{31}$$

Due to the form adopted for the mixing matrix, oscillatory terms will not appear in the transition probability between ν_τ and the other flavors, remaining only the decoherence terms for generating non-zero neutrino conversion related to ν_τ .

The survival and conversion probabilities of interest for the case $2|\Delta_{ij}| \geq |\Delta\gamma_{kl}|$ can be explicitly built now and are given by

$$P_{\nu_e \rightarrow \nu_e}^{\text{D}\oplus\text{O}}(t) = \frac{1}{3} + \frac{1}{2} \sin^2 2\theta \left[\cos \left(\frac{|\Omega_{12}|t}{2} \right) + \frac{\Delta\gamma_{12}}{|\Omega_{12}|} \sin \left(\frac{|\Omega_{12}|t}{2} \right) \right] e^{-\frac{1}{2}(\gamma_1 + \gamma_2)t} \quad (32)$$

$$+ \frac{1}{2} \cos^2 2\theta e^{-\gamma_3 t} + \frac{1}{6} e^{-\gamma_8 t}, \quad (33)$$

$$P_{\nu_e \rightarrow \nu_\mu}^{\text{D}\oplus\text{O}}(t) = \frac{1}{3} - \frac{1}{2} \sin^2 2\theta \left[\cos \left(\frac{|\Omega_{12}|t}{2} \right) + \frac{\Delta\gamma_{12}}{|\Omega_{12}|} \sin \left(\frac{|\Omega_{12}|t}{2} \right) \right] e^{-\frac{1}{2}(\gamma_1 + \gamma_2)t}$$

$$- \frac{1}{2} \cos^2 2\theta e^{-\gamma_3 t} + \frac{1}{6} e^{-\gamma_8 t}, \quad (34)$$

$$P_{\nu_e \rightarrow \nu_\tau}^{\text{D}\oplus\text{O}}(t) = \frac{1}{3} - \frac{1}{3} e^{-\gamma_8 t}, \quad (35)$$

$$P_{\nu_\mu \rightarrow \nu_\mu}^{\text{D}\oplus\text{O}}(t) = \frac{1}{3} + \frac{1}{2} \sin^2 2\theta \left[\cos \left(\frac{|\Omega_{12}|t}{2} \right) + \frac{\Delta\gamma_{12}}{|\Omega_{12}|} \sin \left(\frac{|\Omega_{12}|t}{2} \right) \right] e^{-\frac{1}{2}(\gamma_1 + \gamma_2)t}$$

$$+ \frac{1}{2} \cos^2 2\theta e^{-\gamma_3 t} + \frac{1}{6} e^{-\gamma_8 t}, \quad (36)$$

$$P_{\nu_\mu \rightarrow \nu_\tau}^{\text{D}\oplus\text{O}}(t) = \frac{1}{3} - \frac{1}{3} e^{-\gamma_8 t}, \quad (37)$$

and analogous expressions can be written for the case $2|\Delta_{ij}| < |\Delta\gamma_{kl}|$, through the substitutions $\cos \rightarrow \cosh$, $\sin \rightarrow \sinh$.

Once we have defined these probabilities we will check their consistency with the tendency indicated by the atmospheric neutrino data. Our test will be based upon the definitions for the normalized e -events and μ -events given in the Eqs. (28) and (29).

Since our probabilities involving mixing with ν_e are going to be applied for the atmospheric neutrino data, matter effects must be considered. This is because the atmospheric neutrino data involve neutrinos which travel different distances through the Earth. The inclusion of matter effects in the probability expressions is really straightforward in the case where constant matter density is considered [4]. Basically, we only need to replace $\theta \rightarrow \theta_m$ (in the corresponding sines and cosines), and $\Delta_{13} \rightarrow \Delta_{13}^m$ where

$$\sin^2 2\theta_m(x) = \frac{\sin^2 2\theta}{\sin^2 2\theta + (x - \cos 2\theta)^2}, \quad (38)$$

and,

$$\Delta_{13}^m = \sqrt{\sin^2 2\theta + (x - \cos 2\theta)^2} \Delta_{13}, \quad (39)$$

where x is given by

$$x = \frac{2\sqrt{2} G_F n_e E_\nu}{\Delta m_{13}^2} \simeq 0.76 \frac{\rho(\text{g cm}^{-3}) E_\nu(\text{GeV})}{\Delta m_{13}^2(10^{-4} \text{ eV}^2)}, \quad (40)$$

G_F is the Fermi constant, n_e the electron number density, E_ν the neutrino energy and ρ the matter density.

For evaluating how these matter effects affect our probabilities, we will choose the following values for the relevant parameters: $\Delta m_{13}^2 \leq 3 \times 10^{-4} \text{ eV}^2$, consistent with the large mixing angle (LMA) solution to the solar neutrino problem [27], $E_\nu \sim 10 \text{ GeV}$, consistent with the mean neutrino energy for multi-GeV events, and $\rho(\text{g cm}^{-3}) \simeq 2.75$ [28], this value is one of the lowest values in the Earth matter density profile. Using these values we estimate that $\sin^2 2\theta_m \rightarrow 0$ and $\cos^2 2\theta_m \rightarrow 1$. Note that this will happen even faster for higher values of matter density. As a result, we can write simplified probabilities for this case

$$P_{\nu_e \rightarrow \nu_e}^{\text{D}\oplus\text{O}}(t) \simeq \frac{1}{3} + \frac{1}{2}e^{-\gamma_3 t} + \frac{1}{6}e^{-\gamma_8 t}, \quad (41)$$

$$P_{\nu_\mu \rightarrow \nu_e}^{\text{D}\oplus\text{O}}(t) \simeq \frac{1}{3} - \frac{1}{2}e^{-\gamma_3 t} + \frac{1}{6}e^{-\gamma_8 t}, \quad (42)$$

$$P_{\nu_\mu \rightarrow \nu_\mu}^{\text{D}\oplus\text{O}}(t) \simeq \frac{1}{3} + \frac{1}{2}e^{-\gamma_3 t} + \frac{1}{6}e^{-\gamma_8 t}, \quad (43)$$

here we go one step further and assume $\gamma_8 t \rightarrow 0$,

$$P_{\nu_\mu \rightarrow \nu_\mu}^{\text{D}\oplus\text{O}}(t) = \frac{1}{2} + \frac{1}{2}e^{-\gamma_3 t}, \quad (44)$$

thus we mimic completely the two generation expression for decoherence in $P_{\nu_\mu \rightarrow \nu_\mu}$.

At this point we have $P_{\nu_\mu \rightarrow \nu_\mu} = P_{\nu_e \rightarrow \nu_e} = P$, $P_{\nu_e \rightarrow \nu_\mu} = 1 - P$ and $P_{\nu_e \rightarrow \nu_\tau} = P_{\nu_\mu \rightarrow \nu_\tau} = 0$. Writing Eqs. (28) and (29) as a function of P , we obtain

$$R_\mu = \left(1 - \frac{1}{r}\right) \langle P \rangle + \frac{1}{r}, \quad (45)$$

$$R_e = (1 - r) \langle P \rangle + r. \quad (46)$$

Now we can check, in a model independent way, the consistency of these expressions with the atmospheric neutrino observations. To make an analysis consistent with our assumption on the neutrino energy, we will look at the tendencies of multi-GeV events, this implies $r \approx 3$. The e -like multi-GeV events are consistent with $R_e \simeq 1$ for any neutrino trajectory, so that

$$R_e = -2\langle P \rangle + 3 \simeq 1 \Rightarrow \langle P \rangle \simeq 1, \quad (47)$$

and consequently,

$$R_\mu = \frac{2}{3}\langle P \rangle + \frac{1}{3} \simeq 1, \quad (48)$$

which is in strong disagreement with the behavior of neutrinos coming from below the horizon, since μ -like multi-GeV events are indicating in average that $R_\mu \simeq 0.5 - 0.6$. Therefore this case is also in conflict with the present data.

2. Mixing in the ν_e - ν_τ sector

In this case we turn on only the mixing matrix elements that connect ν_e - ν_τ , which implies the following mixing matrix

$$U = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}, \quad (49)$$

so that the mass induced oscillation terms do not appear in any transition involving ν_μ .

Then, for the case $2|\Delta_{ij}| \geq |\Delta\gamma_{kl}|$, we obtain the probability formulae

$$P_{\nu_e \rightarrow \nu_e}^{\text{D}\oplus\text{O}}(t) = \frac{1}{3} + \frac{1}{2} \cos^4 \theta e^{-\gamma_3 t} + \frac{1}{6} (\cos^2 \theta - 2 \sin^2 \theta)^2 e^{-\gamma_8 t} \quad (50)$$

$$+ \frac{1}{2} \sin^2 2\theta \left[\cos \left(\frac{|\Omega_{13}|t}{2} \right) + \frac{\Delta\gamma_{45}}{|\Omega_{13}|} \sin \left(\frac{|\Omega_{13}|t}{2} \right) \right] e^{-\frac{1}{2}(\gamma_4 + \gamma_5)t}, \quad (51)$$

$$P_{\nu_e \rightarrow \nu_\mu}^{\text{D}\oplus\text{O}}(t) = \frac{1}{3} - \frac{1}{2} \cos^2 \theta e^{-\gamma_3 t} + \frac{1}{6} (\cos^2 \theta - 2 \sin^2 \theta) e^{-\gamma_8 t}, \quad (52)$$

$$P_{\nu_e \rightarrow \nu_\tau}^{\text{D}\oplus\text{O}}(t) = \frac{1}{3} + \frac{1}{2} \cos^2 \theta \sin^2 \theta e^{-\gamma_3 t} + \frac{1}{6} (\cos^2 \theta - 2 \sin^2 \theta) (\sin^2 \theta - 2 \cos^2 \theta) e^{-\gamma_8 t} \quad (53)$$

$$- \frac{1}{2} \sin^2 2\theta \left[\cos \left(\frac{|\Omega_{13}|t}{2} \right) + \frac{\Delta\gamma_{45}}{|\Omega_{13}|} \sin \left(\frac{|\Omega_{13}|t}{2} \right) \right] e^{-\frac{1}{2}(\gamma_4 + \gamma_5)t}, \quad (54)$$

$$P_{\nu_\mu \rightarrow \nu_\mu}^{\text{D}\oplus\text{O}}(t) = \frac{1}{3} + \frac{1}{2} e^{-\gamma_3 t} + \frac{1}{6} e^{-\gamma_8 t}, \quad (55)$$

$$P_{\nu_\mu \rightarrow \nu_\tau}^{\text{D}\oplus\text{O}}(t) = \frac{1}{3} - \frac{1}{2} \sin^2 \theta e^{-\gamma_3 t} + \frac{1}{6} (\sin^2 \theta - 2 \cos^2 \theta)^2 e^{-\gamma_8 t}, \quad (56)$$

and once again, the case $2|\Delta_{ij}| < |\Delta\gamma_{kl}|$ can be obtained just by substituting the harmonic functions by their hyperbolic partners.

Before analyzing the probabilities in this case, we will make some further assumptions. First, we will take $3 \times 10^{-5} \lesssim \Delta m_{13}^2 / \text{eV}^2 \lesssim 19 \times 10^{-5}$ and $0.25 \lesssim \tan^2 \theta \lesssim 0.65$, both consistent with the LMA solution to the solar neutrino problem [27], second we will neglect all the decoherence parameters for the L/E_ν at the range of the atmospheric neutrino scale, with the exception of γ_3 , which is needed in order to make $P_{\nu_\mu \rightarrow \nu_\mu}$ similar to its form in the decoherence solution to the atmospheric neutrino anomaly in two generations.

Once we have accepted these assumptions, we can study the compatibility of these probabilities with the current data. We will not use the atmospheric neutrino data to test the viability of the decoherence solution in the three generations scheme. This is because the probabilities are written as a function of $\cos \theta$ and $\sin \theta$, which makes it difficult to extract correct conclusions without introducing refinements to the qualitative analysis we have used up to now. Instead, we will simply look at CHOOZ data. This will be enough to give us a good idea about the compatibility of

the decoherence solution in the three neutrino scheme, through the observations in the channel $\nu_e \rightarrow \nu_e$. Since we are going to use for this study $P_{\nu_e \rightarrow \nu_e}$, it is convenient to write it down after the assumptions mentioned above are taken into account

$$P_{\nu_e \rightarrow \nu_e}^{\text{D}\oplus\text{O}}(L) = \frac{1}{3} + \frac{1}{2} \left(\cos^4 \theta e^{-\gamma_3^* L/E_\nu} + \sin^2 2\theta \cos(\Delta_{13} L) \right) + \frac{1}{6} (\cos^2 \theta - 2 \sin^2 \theta)^2. \quad (57)$$

We have obtained excluded regions in the plane $(\cos^2 \theta, \gamma_3^*)$, with $\gamma_3 = \gamma_3^*/E_\nu$, following the statistical procedure described in Ref. [5]. We must point out that variations in Δm_{13}^2 will not affect these excluded regions, since $\cos(\Delta_{13} L) \rightarrow 1$ in the CHOOZ range.

The results are shown in Fig. 1, where the points to the right of the contours are excluded. We observe that the point in this plane which corresponds to the value of the decoherence parameter which best explains the atmospheric neutrino data, $\gamma_3^* = 1.2 \times 10^{-21} \text{ GeV}^2$, as well as $\cos^2 \theta$ which corresponds to the best fit value for the LMA solution to the solar neutrino problem, $\cos^2 \theta \approx 0.73$ ($\sin^2 2\theta \approx 0.8$), denoted by a black dot, is excluded at 99 % C.L. In general, if $\cos^2 \theta \gtrsim 0.5$ we can exclude $\gamma_3^* \lesssim 7.0 \times 10^{-22} \text{ GeV}^2$ at 99 % C.L. In this way we observe that CHOOZ data also highly disfavor this three neutrino scheme, since it excludes a large region of γ_3^* compatible with the atmospheric neutrino solution for values of $\cos^2 \theta$ consistent with the LMA solution to the solar neutrino deficit.

V. CONCLUSIONS

Under the assumption that the neutrino system can interact with a pervasive environment, we have obtained neutrino probability formulae for three neutrino generations, taking into account quantum dissipative effects coming from the interaction with the medium on top of the OIM mechanism. The damping terms were brought in through the quantum dynamical semigroup formalism. This approach is very useful, since no *a priori* assumption on the form of neutrino-medium interaction has to be made. Some simplifications of the form of the dissipative matrix were adopted based on results in two generations.

We have performed a qualitative analysis to test if the two generation decoherence solution to the atmospheric neutrino problem viewed in this hybrid three neutrino framework, can still explain the tendencies of the current experimental neutrino data.

We have analyzed two different cases, the first one considering only PD and the second including a mixture of both conversion mechanisms, that is, decoherence plus OIM. The second case was further subdivided into two cases, according to the choice of mixing matrix: i) mass and mixing contributions connecting $\nu_e \rightarrow \nu_\mu$; ii) mass and mixing contribution connecting $\nu_e \rightarrow \nu_\tau$. We have observed that all of these cases are clearly disfavored by recent relevant experimental neutrino data. Particularly,

in the PD case, the fact that $P_{\nu_e \rightarrow \nu_e} = P_{\nu_\mu \rightarrow \nu_\mu}$ is not compatible with the constraint given by CHOOZ combined with SK data (R_e and R_μ) or with K2K results. For the hybrid case of decoherence plus non-null mixing in $\nu_e \rightarrow \nu_\mu$, the same prediction $P_{\nu_e \rightarrow \nu_e} = P_{\nu_\mu \rightarrow \nu_\mu}$ arises which is clearly not supported by SK data. In the case of decoherence plus mixing in $\nu_e \rightarrow \nu_\tau$, we have made a statistical analysis of CHOOZ data using our theoretical expression for $P_{\nu_e \rightarrow \nu_e}$, with some simplifications. We have obtained that the best fit value for the decoherence solution to the atmospheric neutrino problem, $\gamma_3^* = 1.2 \times 10^{-21} \text{ GeV}^2$, is highly disfavored by data. Values of $\gamma_3^* > 3. \times 10^{-22} \text{ GeV}^2$ for $\cos \theta$ consistent with the LMA solution to the solar neutrino deficit are, in general, excluded at 99 % C.L..

Although, the tests we have performed in the three neutrino scheme indicate a disagreement between data and theoretical expectations, this does not mean that dissipative effects can not exist as subleading processes, with the full three neutrino OIM as the main mechanism for neutrino flavor conversion. In fact, the formulae developed here are interesting to be used to help establishing limits on the decoherence parameters or to try to detect their effects using an appropriate three neutrino description.

Also it is worth to stress that we have worked here in a simplified situation in which the dissipative matrix is diagonal. The presence of off-diagonal terms can certainly produce interesting effects on the probability expressions. Our formulae can be easily modified to include these off-diagonal terms. However, any further qualitative or quantitative analysis in three generations will not be so direct.

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APPENDIX A: EXPLICIT FORM OF \mathbf{M}

We present here the explicit form of the matrix \mathbf{M} , its eigenvalues and eigenvectors, as well as the diagonalizing matrix \mathbf{D} and its inverse defined in Eq. (12). Starting from the definition of \mathbf{M} in Eq. (9), taking as valid the approximation $H_{\text{eff}} \sim H$ [4], we can write

$$\mathbf{M} = \begin{pmatrix} -\gamma_1 & -\Delta_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta_{12} & -\gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\gamma_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma_4 & -\Delta_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_{13} & -\gamma_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma_6 & -\Delta_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta_{23} & -\gamma_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma_8 \end{pmatrix}, \quad (\text{A1})$$

with the definitions

$$h_3 = \Delta_{12}, \quad h_8 = \frac{1}{\sqrt{3}}(\Delta_{13} + \Delta_{23}) \quad \text{and} \quad \Delta_{ij} = \frac{\Delta m_{ij}^2}{2p}, \quad i, j = 1, 2, 3. \quad (\text{A2})$$

Now solving the secular equation $\det(\mathbf{M} - \lambda \mathbf{1}) = 0$, we get

$$\begin{aligned} \lambda_1 &= \frac{1}{2} \left[-(\gamma_1 + \gamma_2) - \sqrt{(\gamma_2 - \gamma_1)^2 - 4\Delta_{12}^2} \right] = \frac{1}{2} [-(\gamma_1 + \gamma_2) - \Omega_{12}] \\ \lambda_2 &= \frac{1}{2} \left[-(\gamma_1 + \gamma_2) + \sqrt{(\gamma_2 - \gamma_1)^2 - 4\Delta_{12}^2} \right] = \frac{1}{2} [-(\gamma_1 + \gamma_2) + \Omega_{12}] \\ \lambda_3 &= -\gamma_3 \\ \lambda_4 &= \frac{1}{2} \left[-(\gamma_4 + \gamma_5) - \sqrt{(\gamma_5 - \gamma_4)^2 - 4\Delta_{13}^2} \right] = \frac{1}{2} [-(\gamma_4 + \gamma_5) - \Omega_{13}] \\ \lambda_5 &= \frac{1}{2} \left[-(\gamma_4 + \gamma_5) + \sqrt{(\gamma_5 - \gamma_4)^2 - 4\Delta_{13}^2} \right] = \frac{1}{2} [-(\gamma_4 + \gamma_5) + \Omega_{13}] \\ \lambda_6 &= \frac{1}{2} \left[-(\gamma_6 + \gamma_7) - \sqrt{(\gamma_7 - \gamma_6)^2 - 4\Delta_{23}^2} \right] = \frac{1}{2} [-(\gamma_6 + \gamma_7) - \Omega_{23}] \\ \lambda_7 &= \frac{1}{2} \left[-(\gamma_6 + \gamma_7) + \sqrt{(\gamma_7 - \gamma_6)^2 - 4\Delta_{23}^2} \right] = \frac{1}{2} [-(\gamma_6 + \gamma_7) + \Omega_{23}] \\ \lambda_8 &= -\gamma_8 \end{aligned} \quad (\text{A3})$$

The associated eigenvectors are in turn

$$\begin{aligned} \mathbf{v}_1^T &= \left(\frac{\lambda_1 + \gamma_2}{\Delta_{12}}, 1, 0, 0, 0, 0, 0, 0 \right) \\ \mathbf{v}_2^T &= \left(\frac{\lambda_2 + \gamma_2}{\Delta_{12}}, 1, 0, 0, 0, 0, 0, 0 \right) \\ \mathbf{v}_3^T &= (0, 0, 1, 0, 0, 0, 0, 0) \end{aligned}$$

$$\begin{aligned}
\mathbf{v}_4^T &= \left(0, 0, 0, \frac{\lambda_4 + \gamma_5}{\Delta_{13}}, 1, 0, 0, 0\right) \\
\mathbf{v}_5^T &= \left(0, 0, 0, \frac{\lambda_4 + \gamma_5}{\Delta_{13}}, 1, 0, 0, 0\right) \\
\mathbf{v}_6^T &= \left(0, 0, 0, 0, 0, \frac{\lambda_6 + \gamma_7}{\Delta_{23}}, 1, 0\right) \\
\mathbf{v}_7^T &= \left(0, 0, 0, 0, 0, \frac{\lambda_7 + \gamma_7}{\Delta_{23}}, 1, 0\right) \\
\mathbf{v}_8^T &= (0, 0, 0, 0, 0, 0, 0, 1),
\end{aligned} \tag{A4}$$

where the superscript T denotes transposition. One can now construct the diagonalizing matrix \mathbf{D} and its inverse

$$\mathbf{D} = \begin{pmatrix} \frac{\lambda_1 + \gamma_2}{\Delta_{12}} & \frac{\lambda_2 + \gamma_2}{\Delta_{12}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\lambda_4 + \gamma_5}{\Delta_{13}} & \frac{\lambda_5 + \gamma_5}{\Delta_{13}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\lambda_6 + \gamma_7}{\Delta_{23}} & \frac{\lambda_7 + \gamma_7}{\Delta_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{A5}$$

$$\mathbf{D}^{-1} = \begin{pmatrix} -\frac{\Delta_{12}}{\Omega_{12}} & \frac{\lambda_2 + \gamma_2}{\Omega_{12}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\Delta_{12}}{\Omega_{12}} & -\frac{\lambda_1 + \gamma_2}{\Omega_{12}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\Delta_{13}}{\Omega_{13}} & \frac{\lambda_5 + \gamma_5}{\Omega_{13}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\Delta_{13}}{\Omega_{13}} & -\frac{\lambda_4 + \gamma_5}{\Omega_{13}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\Delta_{23}}{\Omega_{23}} & \frac{\lambda_7 + \gamma_7}{\Omega_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\Delta_{23}}{\Omega_{23}} & -\frac{\lambda_6 + \gamma_7}{\Omega_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{A6}$$

APPENDIX B: EXPLICIT FORM OF THE COEFFICIENTS ρ_μ^α

Flavor eigenstates $|\nu_\alpha\rangle$, $\alpha = e, \mu, \tau$ can be written as a function of mass eigenstates $|\nu_k\rangle$, $k = 1, 2, 3$ through a unitary matrix U

$$|\nu_\alpha\rangle = \sum_{k=1}^3 U_{\alpha k}^* |\nu_k\rangle \quad \text{and} \quad \sum_{k=1}^3 U_{k\beta} U_{k\alpha}^* = \sum_{k=1}^3 U_{\beta k} U_{\alpha k}^* = \delta_{\alpha\beta}, \quad (\text{B1})$$

we can express the density matrix of a flavor state $|\nu_\alpha\rangle$ as

$$\begin{aligned} \rho^\alpha &= |\nu_\alpha\rangle\langle\nu_\alpha| = \left(\sum_{k=1}^3 U_{\alpha k}^* |\nu_k\rangle \right) \left(\sum_{l=1}^3 U_{\alpha l} \langle\nu_l| \right) \\ &= \sum_{k,l=1}^3 U_{\alpha k}^* U_{\alpha l} |\nu_k\rangle\langle\nu_l|. \end{aligned}$$

In the mass eigenstates basis we have

$$\langle\nu_m|\rho^\alpha|\nu_n\rangle = \sum_{k,l=1}^3 U_{\alpha k}^* U_{\alpha l} \langle\nu_m|\nu_k\rangle\langle\nu_l|\nu_n\rangle = U_{\alpha m}^* U_{\alpha n}, \quad (\text{B2})$$

so that

$$[\rho^\alpha] = \begin{pmatrix} |U_{\alpha 1}|^2 & U_{\alpha 1}^* U_{\alpha 2} & U_{\alpha 1}^* U_{\alpha 3} \\ U_{\alpha 2}^* U_{\alpha 1} & |U_{\alpha 2}|^2 & U_{\alpha 2}^* U_{\alpha 3} \\ U_{\alpha 3}^* U_{\alpha 1} & U_{\alpha 3}^* U_{\alpha 2} & |U_{\alpha 3}|^2 \end{pmatrix}. \quad (\text{B3})$$

Therefore, the coefficients $\rho_\mu^\alpha = 2\text{Tr}[\rho^\alpha F_\mu]$ can be explicitly written as

$$\begin{aligned} \rho_0^\alpha &= \sqrt{2/3} \\ \rho_1^\alpha &= 2 \text{Re}(U_{\alpha 1}^* U_{\alpha 2}) \\ \rho_2^\alpha &= -2 \text{Im}(U_{\alpha 1}^* U_{\alpha 2}) \\ \rho_3^\alpha &= |U_{\alpha 1}|^2 - |U_{\alpha 2}|^2 \\ \rho_4^\alpha &= 2 \text{Re}(U_{\alpha 1}^* U_{\alpha 3}) \\ \rho_5^\alpha &= -2 \text{Im}(U_{\alpha 1}^* U_{\alpha 3}) \\ \rho_6^\alpha &= 2 \text{Re}(U_{\alpha 2}^* U_{\alpha 3}) \\ \rho_7^\alpha &= -2 \text{Im}(U_{\alpha 2}^* U_{\alpha 3}) \\ \rho_8^\alpha &= \frac{1}{\sqrt{3}}(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 - 2|U_{\alpha 3}|^2) \end{aligned} \quad (\text{B4})$$

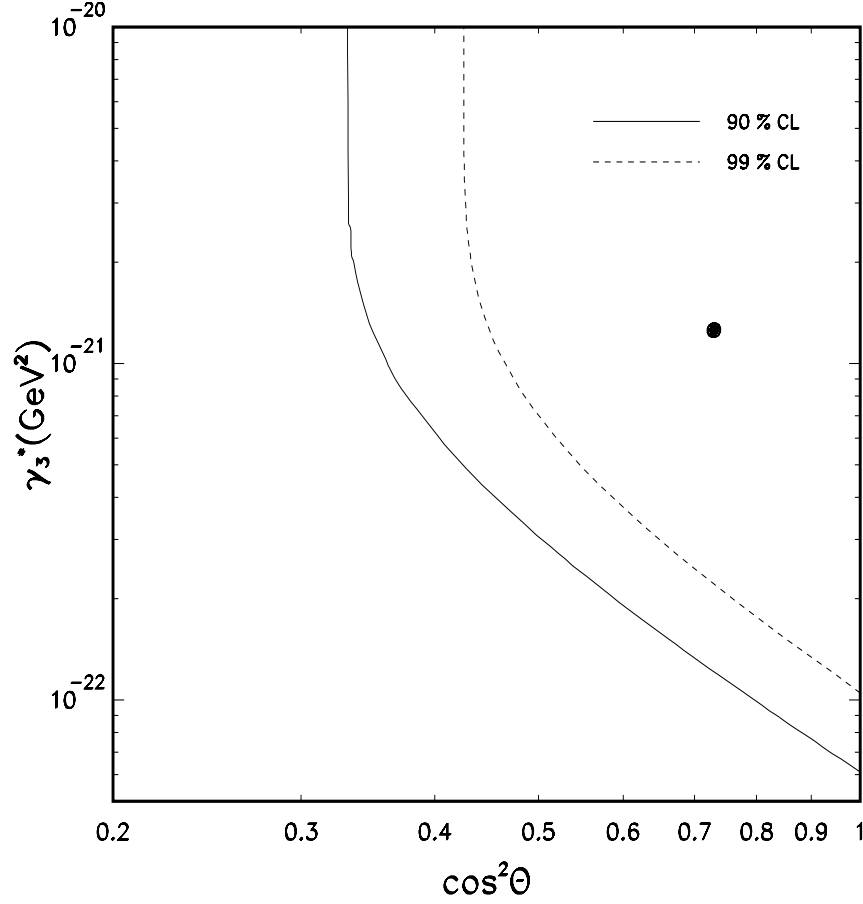


FIG. 1. Regions in the plane $(\cos^2 \theta, \gamma_3^*)$ excluded by CHOOZ data. The black point denotes the best fit value for the decoherence solution to the atmospheric neutrino problem, $\gamma_3^* = 1.2 \times 10^{21} \text{ GeV}^2$ as well as for the LMA solution to the solar neutrino one, $\cos^2 \theta \approx 0.73$. All points to the right of the curves are excluded at 90 % (continuous line) and 99 % (dashed line) C.L.